Critical behavior of a nonequilibrium system with two nonordering conserved fields

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We investigate the critical behavior of a nonequilibrium system with two particle species A and B that exhibits a continuous absorbing-state phase transition. The number of particles of each species (N_A and N_B) is conserved by the dynamical process. Numerical results show that the order parameter exponent β depends on the ratio N_B/N_A at criticality. Some aspects of critical dynamic behavior are also studied, namely, the decay of the active density at criticality and the critical spreading of a perturbation to an absorbing configuration. Anomalies in the relaxation are associated with the presence of different time scales in the dynamics of the model.

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A considerable number of nonequilibrium models exhibiting phase transitions from active to absorbing phases have been investigated over recent years [1]. These studies give strong support to the conjecture made by Grassberger [2] that continuous transitions to a unique absorbing state fall generically in the universality class of directed percolation (DP), provided there are no additional symmetries or conservation laws and only short range interactions are present. The static critical behavior of various models with an infinite number of absorbing states also falls in the DP universality class.

Nonequilibrium models with extra symmetries or conserved quantities, and indeed out of DP, have also been investigated. Models where the parity in the number of particles is conserved [3–5] or with a complete symmetry among absorbing states [6], as well as models where both features are present [7], all fall in the parity conserving universality class. More recently, models where one quantity (the total number of particles) is conserved led to the identification of other universality classes: clearly, conserved lattice gases [8], stochastic sandpile models with conserved energy [9,10], and the nondiffusive limit of the two-species reaction-diffusion model [11] all fall in the same universality class; different critical behavior is expected, however, if, in the last model, both types of particle are allowed to diffuse [12–14].

In this work we consider a nonequilibrium system where two quantities are conserved. The model is defined such that each site of a one-dimensional (1D) lattice can be in one of four states: empty, occupied either by one A or one B particle, or doubly occupied by one A and one B particle. Two particles of the same species are not allowed to occupy the same site. The site is considered active if it is occupied by a pair (AB) of particles. Once an active site has been selected (sequentially), the dynamics proceeds as follows. If each neighbor is occupied by a particle of a different species, if both neighbors are empty, or if one is empty and the other one occupied by one particle, then the active site becomes vacant and each neighbor receives one of its particles. If both neighbors are active, no transfer can occur; otherwise, there is the transfer of just one particle from the active site to the allowed neighbor (if there are two different possibilities, each one is chosen randomly with probability 0.5). Thus, when both neighbors of the active site are empty, or when one is empty and one is active, the number of active sites decreases by 1, whereas it increases by 1 if each neighbor is occupied by a particle of a different species; in the other cases, there is simply a redistribution of vacant and doubly and singly occupied sites in the neighborhood. It is obvious that the numbers of particles of each species are conserved by the dynamics. This model can be considered a two-species generalization of a conserved threshold transfer process [8,15].

We performed numerical simulations of the stationary properties of the model. System sizes varied between $L=200$ and $L=4000$, the number of time steps between $t=2\times10^4$ and $t=2\times10^6$ (closest to the critical point), and we averaged over 200 to 2000 independent samples. The initial configuration was generated by randomly distributing the (N_A) A particles and the (N_B) B particles among the L sites of the lattice, and periodic boundary conditions were used. For a fixed N_A and a small N_B ($<L-N_A$), there are no doubly occupied sites in the stationary state and the system cannot evolve from there; this corresponds to the inactive phase. As we increase N_B, the system undergoes a phase transition to an active phase, where the concentration of doubly occupied sites (the order parameter ρ_{AB}) is nonzero. In Fig. 1 we show ρ_{AB} as a function of N_B for $L=2000$ and different values of N_A.

As can be seen, the transition occurs at $N_B=L-N_A$. The long relaxation times that appear, as one might expect, very close to the critical point can be associated with configurations like $A\ A\ 0\ AB\ B\ B\ B\ AB\ 0\ B\ B$, where active sites have one empty neighbor. If one waits long enough, one is left with just one of these active sites with one empty neighbor. Once periodic boundary conditions are being used, this applies also to configurations like $A\ A\ AB\ 0\ B\ B\ AB\ 0\ B\ B$ where the distance between the two active sites fluctuates around the initial value in a random walk way and therefore the two active sites can be close to each other and coalesce into one active site after a sufficiently long time. For $N_B=L-N_A$, and except for the case

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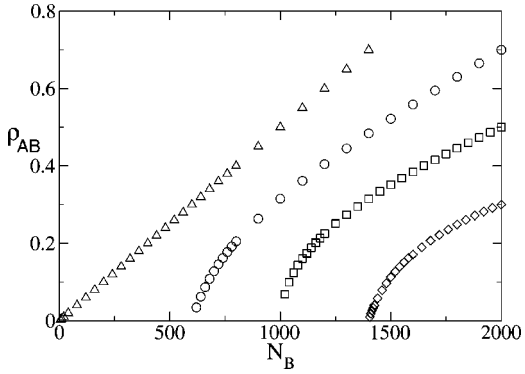

FIG. 1. ρ_{AB} as a function of N_B for $L=2000$ and different values of N_A: $N_A=2000$ (△), 1400 (○), 1000 (□), 600 (◇).

of an initial configuration with no doubly occupied and no empty sites (which occurs with a very small probability), the concentration of doubly occupied sites at stationarity is then $\rho_{AB}^{(c)}=1/L$, and becomes zero in the thermodynamic limit.

When $N_A=L$, any B particle that is being added leads to an active site (which, given the prescribed dynamics, will move around in a random walk way) and there is no possibility of having an empty site; therefore ρ_{AB} grows linearly with N_B. The situation is different for $N_A\neq L$. For a clear comparison of the critical behavior that occurs for different values of N_A, we rescaled variables as $y=\rho_{AB}/(N_A/L)$, $x=(N_A+N_B-L)/N_A$, so that the transition always occurs at $x_c=0$ and $y=1$ when $N_B=L$; in Fig. 2 we have plotted y vs x for the same data used in Fig. 1. As can be seen, the order parameter follows a power law $y\sim x^{\beta}$, but β is a function of N_A. In order to extract this exponent we have used a log-log plot (as represented in Fig. 3); this yields $\beta=0.82(1)$ and $\beta=0.50(1)$ for $N_A=600$ and $N_A=1000$, respectively. Although the curves y vs x for $N_A=600$ and $N_A=1400$ do not coincide, the exponent β is the same, within numerical accuracy. As a matter of fact the dynamics is completely symmetric under the exchange of A and B particles, so one expects the same behavior right at criticality, for $N_A=600$, $N_B=1400$ and for $N_A=1400$, $N_B=600$; although β describes the off-critical singularity, it is indeed plausible that it should be the same in both cases. Accordingly, $\beta(\lambda)$, where $\lambda=(N_A/L)$ $[\lambda=1/(1+N_B/N_A)$ at criticality] should

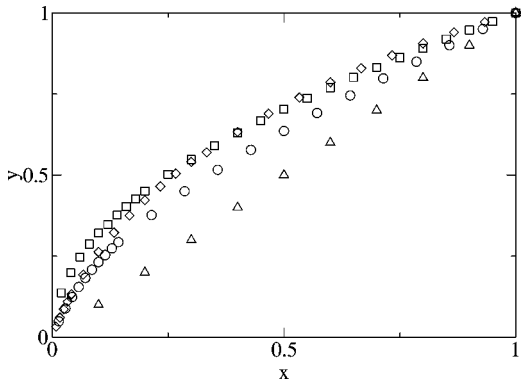

FIG. 2. The same data as in Fig. 1, with $y=\rho_{AB}/(N_A/L)$, $x=(N_A+N_B-L)/N_A$.

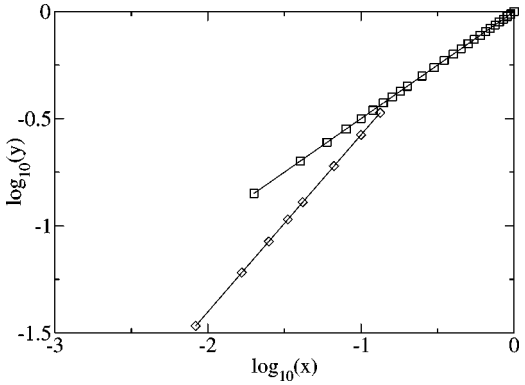

FIG. 3. Log-log plot of y vs x for $\lambda=0.5$ (□), $\lambda=0.3$ (◇). The straight lines are least squares linear fits (slopes of 0.50 and 0.82, respectively).

be symmetric around 0.5; of course β cannot be defined for $\lambda=0$ since in the absence of A particles there is no active phase at all.

According to the finite size scaling ansatz the stationary density of active sites at the critical point follows a power-law behavior $\rho_{AB}^{(c)}\sim L^{-\beta/\nu_{\perp}}$, where $\nu_{\perp}$ is the exponent that characterizes the divergence of the correlation length. This gives $\beta/\nu_{\perp}=1$ for all λ.

We then considered time dependent quantities. First we studied $\rho_{AB}(t)$ at criticality, using homogeneous initial configurations, as before. In Fig. 4, we show a log-log plot of $\rho_{AB}(t)$ vs t for $\lambda=0.3$ and different system sizes.

The short-time decay of the order parameter density follows a power law $\rho_{AB}\sim t^{-\theta'}$ $[\theta'=0.209(5)]$ over a few decades; since this is independent of the size, it is probably associated with a purely local process. For longer times, $\rho_{AB}(t)$ exhibits a sharp drop and enters a second power-law regime $\rho_{AB}\sim t^{-\theta}$ $[\theta=0.49(1)]$ before the final approach to the steady state.

The relaxation times $\tau(L)$ at criticality can be obtained from the linear portion of a semilogarithmic plot of the excess density $\rho_{AB}(t)-\rho_{AB}^{(c)}$. These relaxation times are expected to scale as $\tau(L)\sim L^{z}$. The present data are consistent

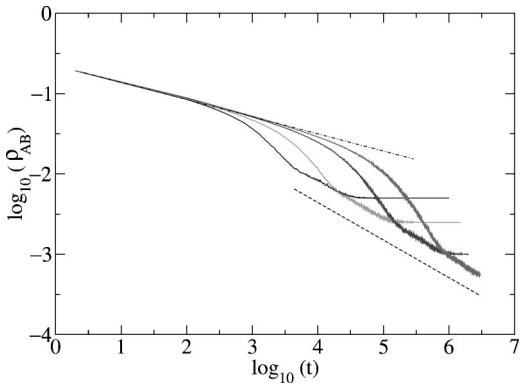

FIG. 4. Relaxation of the active site density at criticality, for $\lambda=0.3$ (from top to bottom, $L=2000,1000,400,200$). The straight line −−−−− has slope $\theta=0.49$. The straight line −·−·−·− has slope 0.209.

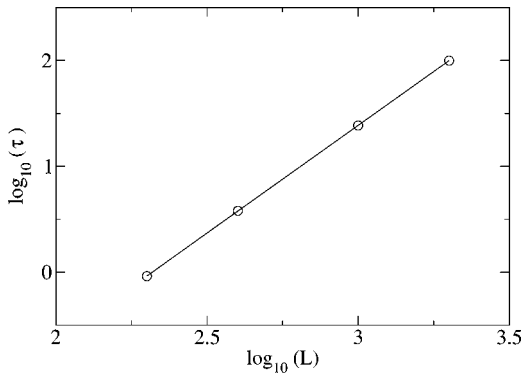

FIG. 5. Log-log plot of relaxation times τ vs L. The straight line is a least squares linear fit; its slope is 2.02.

with $z=2$ (see Fig. 5); however, delimiting the linear part in the semilogarithmic plot is a source of numerical error. Also, the above estimate for θ is consistent with the scaling relation $\theta=\beta/\nu_{\|}=\beta/\nu_{\perp}z$, so the second power-law regime is likely to describe the asymptotic evolution of the order parameter density. A more detailed study of the effects of initial conditions, better statistics, and more extensive data are required in order to establish whether the above exponents are affected by the details of the initial preparation.

Different scaling regimes have been found previously in models with just one kind of particle (and one conserved quantity, the total number of these particles) [9,10]. In the 1D fixed energy version of the Manna stochastic sandpile model studied in [10], the initial dynamics is dominated by relaxation of the initial grain profile, whereas some slow relaxation is likely associated with long-wavelength density variations. In the present model there are two types of particle and more complex relaxation processes can be expected. However, the long-time dynamics appears to be controlled by random walk processes. These are indeed present wherever an active particle is surrounded by particles of the same species, *B B AB B B B* or *A A AB A A*; also, the process by which two active sites reduce to one is random walk like, as explained above. In some models where coalescence or pair annihilation are present the inactive phase is critical and in the universality class of diffusion-annihilation [16]; however the transition between active and inactive phases is characterized by another set of critical exponents. Critical behavior characterized by the exponents $\theta=0.5, z=2, \beta=\nu_{\perp}$ continuously varying with a parameter s has been found by Lipowski in a different model with continuous bond variables [17].

Additional information on the critical dynamics of systems with absorbing states is usually provided by critical spreading; in such experiments, one studies the time evolution of a small perturbation to an otherwise absorbing configuration. In each spreading realization, the quantities that are usually measured are the total activity averaged over all runs $n(t)$, the survival probability $P(t)$, and the linear size over which the perturbation has spread $l(t)$. At the critical point, these quantities have a singular behavior $n(t)\sim t^{\eta'}$, $P(t)\sim t^{-\delta'}$, and $l(t)\sim t^{1/z'}$. In systems with an infinite number of absorbing states, these exponents usually depend on the initial configuration, but satisfy the hyperscaling relation $\eta'+\delta'+\theta=d/z'$ [15].

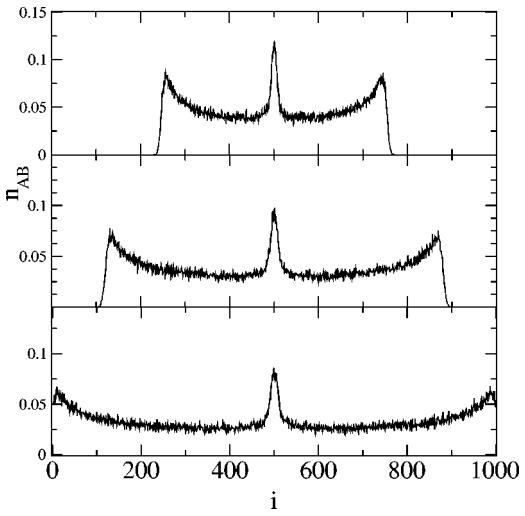

FIG. 6. Histogram of the distribution of active sites for the spreading of a perturbation at the center of the lattice ($L=80\,000$) (from top to bottom, $t=2000,3000,4000$). $n_{AB}(i)$ is the number of active sites in the interval $[80(i-1)+1,80i]$. Averages were done over 5000 independent samples.

We have performed spreading experiments for $\lambda=0.3$. The starting configuration was obtained by randomly distributing N_A particles A among the L sites of a chain, and filling the empty sites with B particles; then at the center of such a configuration we placed a doubly occupied site. Averages were done over a set of 5000 independent trials. In Fig. 6 we show a histogram of the distribution of active sites obtained at three different stages. Clearly, this distribution is far from uniform in the region of size l over which the perturbation has spread. l is approximately linear in t; this can be associated with configurations like *B B 0 AB A A A*, where the active site moves to the right each time it is selected. On the other hand, we cannot expect the hyperscaling relation above (based on the assumption that deep inside the cluster the active site concentration approaches the steady state concentration) to be obeyed.

A more detailed investigation of all the consequences of different choices of initial configurations is needed. Despite the randomness present in the dynamics of the model, the question of ergodicity (often associated with hidden conservation laws) deserves further study. Indeed, we have an indication that time averages from certain initial configurations do not coincide with averages over a large number of trials; whether the fraction of such initial configurations vanishes in the thermodynamic limit or whether they involve extremely large relaxation times to reach the steady state needs to be clarified.

A field theory approach to describe nonequilibrium systems with two conserved fields would be most desirable, in view of the critical behavior found in this system.

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